Overview

Many social and biological systems are hierarchical:
- Social ranks in animal groups.
- Perceived quality of universities and departments.
- Tiers of political candidates; athletes; movie stars...

We introduce a tractable, coevolving network model of hierarchy. This model...
- ...possesses distinct egalitarian and hierarchical regimes, separated by a phase transition.
- ...allows tractable inference from data.
- ...highlights inferred timescales and prestige preferences in real-world systems!

Model Definition: Prestige-Driven Endorsements

A first-order Markov chain model with state \( \mathbf{A} \in \mathbb{R}^{n \times n} \). In each time step:
- Compute ranks \( \mathbf{\gamma} \in \mathbb{R}^n \) from \( \mathbf{A} \). We use SoftMax SpringRank [2] with inverse temperature \( \beta \) (prestige preference).
- Uniformly random agent \( I \) endorses agent \( J \sim \text{Categorical}(\mathbf{\gamma}) \).
- Update: for memory parameter \( \lambda \in \mathbb{R} \),
  \[
  \mathbf{A}' = \lambda \mathbf{A} + (1 - \lambda) \mathbf{E}_{IJ} .
  \]

Model Dynamics Depend on \( \beta \)

Larger \( \beta \) \( \rightarrow \) more stratified ranks.

Phase Transition in the Long-Memory Limit

Theorem: The egalitarian regime \( \mathbf{\gamma}_{\text{eq}} = n^{-1} \mathbf{e} \) is linearly stable in expectation as \( \lambda \rightarrow 1 \) if and only if \( \beta < 2 \).

Maximum-Likelihood Inference

We can estimate the prestige-sensitivity \( \beta \) and memory \( \lambda \) from empirical data via maximum-likelihood. Let \( \mathbf{k} = \mathbf{e}^\mathbf{T} \Delta(\mathbf{l}) \). The log-likelihood is

\[
L(\lambda, \beta; \{\mathbf{A}^{(t)}\}) = \beta \sum_{i=1}^{n} k_i s_i - (\mathbf{k}^\mathbf{T} \mathbf{e}) \log \left( \sum_{i=1}^{n} e^{\beta k_i} \right) + C .
\]

We maximize this expression with respect to \( \lambda \) and \( \beta \). We estimate uncertainty by inverting the observed Fisher information matrix.

Discussion

We have developed a novel model of emergent hierarchy in social networks.
- Model does not assume fixed “fitness” or “quality” for agents.
- Model gives interpretable time-dependent importance (centrality) scores with direct dynamical interpretations.

Future directions include analysis of the non-egalitarian regime and more complex generating mechanisms.

References